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Cheapest superstrategies without the optional decomposition

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Abstract: We follow very closely Föllmer and Kabanov Lagrange multiplier approach to superstrategies in perfect incomplete markets, except that we provide a very simple proof of the existence of a minimizing multiplier in case of a European option under the assumption that the discounted process of the underlying is an $L^2(P)$ martingale for some probability P. Even if it gives the existence of a superstrategy associated to the supremum of the expectations under the equivalent martingale measures, our result is much weaker than the optional decomposition theorem.

Key-words: Superstrategies, Incomplete markets

(Résumé : tsvp)

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Surstratégies minimales sans décomposition optionnelle

Résumé : On propose ici une légère modification de l'approche multiplicateur de Lagrange de Föllmer et Kabanov des surstratégies en marchés incomplets, qui donne une preuve très simple de l'existence d'une surcouverture associée au supremum des espérances sous les mesures martingales équivalentes. Notre résultat est beuacoup plus faible que la décomposition optionnelle.

Mots-clé : Surstratégies, Marchés incomplets

1 Introduction

We tackle in this paper the following very classical issue: consider on a filtered probability space $\left(\Omega, (F), (F_t)_{0\leq t\leq T}, Q\right)$ where $T > 0$ is a fixed deterministic horizon some F_T-measurable random variable Y_T, which stands for the payoff of some European contingent claim with maturity T on an asset with discounted price modeled by an adapted process $(S_t)_{0\leq t\leq T}$ with values in $\mathbb{R}_*^+$ (for clarity's sake we work in a one-dimensional framework).

We shall introduce the following assumption:

$$Y_T \text{ is positive and bounded, the initial } \sigma\text{-field } F_0 \text{ is } Q\text{-trivial} \qquad (H0)$$

Assume moreover that S is a $P-(F_t)$ martingale for some probability P equivalent to Q. Then the Absence of Arbitrage Opportunities is fulfilled, in the sense that there is no predictable process Δ such that $\int_0^T \Delta_t^2 d[S]_t$ is finite almost surely, with the constraint that the process $t \mapsto \int_0^t \Delta_u dS_u$ is bounded from below (we shall say in the sequel that such a process is "admissible"), which satisfies $\int_0^T \Delta_t dS_t \geq 0$-except $\Delta = 0$.

The following program makes sense for a totally risk averse seller of the option (especially in case the only assets available on the market are the underlying and the cash account): find c_* such that

$$c_* = \inf\left\{c \in \mathbb{R} \,/\, \exists\, \Delta \text{ admissible, } c + \int_0^T \Delta_t dS_t \geq Y_T \ a.s.\right\}$$

This program has received much attention in the past few years. The main result is a duality property between the admissible strategies and the set of equivalent martingale measures:

$$M_e(Q) = \{R \sim Q,\ S \text{ is a } R-(F_t) \text{ martingale}\}$$

which leads to:

Theorem 1 *The infimum c_* is attained and*

$$c_* = \sup_{R\in M_e(Q)} E^R[Y_T] \qquad (1)$$

This has been shown first in case of continuous processes by ElKaroui-Quenez ([ElKQ]), later on for locally bounded processes in [K]. A very natural and interesting proof based on a Lagrange multiplier approach has been provided by Föllmer-Kabanov in [FK] where they also drop the local boundedness assumption. This is intimately related to the beautiful characterization of the stochastic integrals among the set of local martingales by Ansel-Stricker in [AS] and Jacka in [J], which extends the classical representation property of Jacod and Yor (cf [RY]): the stochastic integrals are exactly the local martingales which remain local martingales under a change of equivalent martingale measure.

In fact all these papers prove a much deeper result than (1), the celebrated optional decomposition theorem, which can be stated in our framework as follows:

Theorem 2 *Let Z a process which is a positive supermartingale for every R in $M_e(Q)$. Then there is an admissible process Δ and a non-decreasing optional process A with $A_0 = 0$ such that for every t between 0 and T*

$$Z_t = Z_0 + \int_0^t \Delta_u dS_u - A_t \ a.s.$$

Notice that (1) follows quite easily by considering the process

$$Z_t = ess \sup_{M_e(Q)} E\left[Y_T \mid F_t\right]$$

In this paper, we shall directly get (1) by elementary Hilbert space techniques under the assumption

$$(S_t)_{0\le t\le T} \text{ is an } L^2(P) - \text{martingale for some } P \text{ in } M_e(Q) \qquad (H1)$$

by following very closely Föllmer and Kabanov approach.

In the last part we discuss the relationship between theorems 1 and 2. In particular we show that in a situation where the optional decomposition is unique, there may be plenty of admissible processes associated to the minimizing price c^*.

In the rest of the paper, we work under the assumptions $(H0)$ and $(H1)$. P stands for a fixed probability under which $(H1)$ holds.

2 Föllmer and Kabanov Lagrange multiplier approach

2.1 Some notations

Let us introduce the space $\mathcal{H} = \mathcal{L}^2(P)$ of (equivalent classes) of predictable processes $(\lambda_t)_{0\leq t\leq T}$ such that

$$E^P\left[\int_0^T \lambda_t^2 d[S]_t\right] < \infty$$

which we endow with its natural Hilbert space structure.

Since S is an $L^2(P)-$ martingale, $\mathcal{H}$ contains the bounded predictable processes.

Let also $\mathcal{P}$ denote the set of probabilities R equivalent to P such that $\frac{dR}{dP}$ belongs to $L^2(P)$ and $\mathcal{M}$ the subset of $\mathcal{P}$ which consists of martingale measures.

To any R in $\mathcal{P}$ we associate the $P-$martingale $D(R)_t = E^P\left[\frac{dR}{dP} \mid F_t\right]$. As an $L^2(P)-$ martingale $D(R)$ admits a unique decomposition like:

$$D(R)_t = 1 + \int_0^t \lambda(R)_u dS_u + \theta(R)_t$$

where $\theta(R)$ is an $L^2(P)-$ martingale which belongs to the orthogonal subspace of the subspace generated by the processes $t \mapsto \int_0^t \beta_u dS_u$ where β runs into $\mathcal{H}$. We shall denote this subspace par Θ. Moreover $\lambda(R)$ belongs to $\mathcal{H}$.

2.2 The Lagrange multiplier approach

We rephrase in an adequate way the approach followed in [FK].

Consider the quantity

$$A = \sup_{\mathcal{M}} E^R[Y_T]$$

as a constrained optimization problem over $\mathcal{P}$. Then it is tempting to introduce the corresponding Lagrange multiplier:

Lemma 3 *The following holds:*

$$A = \sup_{\mathcal{P}} \inf_{\mathcal{H}} E^R \left[Y_T - \int_0^T \lambda_t dS_t \right]$$

Observe first that the expectation $E^R \left[\int_0^T \lambda_t dS_t \right]$ makes sense, since

$$\begin{aligned} E^R \left[\left| \int_0^T \lambda_t dS_t \right| \right]^2 &= E^P \left[\frac{dR}{dP} \left| \int_0^T \lambda_t dS_t \right| \right]^2 \\ &\leq E^P \left[\left(\frac{dR}{dP} \right)^2 \right] E^P \left[\left(\int_0^T \lambda_t dS_t \right)^2 \right] \end{aligned}$$

by the Cauchy-Schwartz inequality. This also shows that $t \mapsto \int_0^t \lambda_u dS_u$ is a true $R-$martingale for R in $\mathcal{M}$. If R does not belong to $\mathcal{M}$, then for some λ, $E^R \left[\int_0^T \lambda_t dS_t \right] \neq 0$. By considering $\int_0^T (\pm n \lambda_t) \, dS_t$ for large n we get $\inf_{\mathcal{H}} E^R \left[Y_T - \int_0^T \lambda_t dS_t \right] = -\infty$. Therefore

$$\begin{aligned} \sup_{\mathcal{P}} \inf_{\mathcal{H}} E^R \left[Y_T - \int_0^T \lambda_t dS_t \right] &= \sup_{\mathcal{M}} \inf_{\mathcal{H}} E^R \left[Y_T - \int_0^T \lambda_t dS_t \right] \\ &= \sup_{\mathcal{M}} \inf_{\mathcal{H}} E^R [Y_T] = \sup_{\mathcal{M}} E^R [Y_T] = A \end{aligned}$$

The relationship between the unconstrained problem and the superstrategy feature is given by the following:

Lemma 4 *A process λ^* in $\mathcal{H}$ satisfy*

$$\sup_{\mathcal{P}} \inf_{\mathcal{H}} E^R \left[Y_T - \int_0^T \lambda_t dS_t \right] = \sup_{\mathcal{P}} E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right] \tag{2}$$

if and only if

$$A + \int_0^T \lambda_t^* dS_t - Y_T \geq 0 \ a.s. \tag{3}$$

Moreover such a λ^* is admissible.

Assume (2). Thanks to the previous lemma:

$$A = \sup_{\mathcal{P}} E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right]$$

so that for any R in $\mathcal{P}$, $E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right] \leq A$ or yet

$$E^R \left[A + \int_0^T \lambda_t^* dS_t - Y_T \right] \geq 0$$

Let now a measurable set B on which $A + \int_0^T \lambda_t^* dS_t - Y_T < 0$. Consider the probability, for a small $\varepsilon > 0$

$$dQ_\varepsilon = \frac{(\varepsilon + 1_B)\, dP}{\varepsilon + P(B)}$$

then obviously $Q_\varepsilon \in \mathcal{P}$. Now if $P(B) > 0$, $E^{Q_\varepsilon} \left[A + \int_0^T \lambda_t^* dS_t - Y_T \right]$ goes to $\frac{E^P\left[1_B\left(A + \int_0^T \lambda_t^* dS_t - Y_T\right)\right]}{P(B)}$ which is strictly negative. But this is impossible since $E^{Q_\varepsilon} \left[A + \int_0^T \lambda_t^* dS_t - Y_T \right] \geq 0$ for any $\varepsilon > 0$.

Assume (3). Then for every R in $\mathcal{P}$:

$$A \geq E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right]$$

with also by definition $A = \sup_{\mathcal{M}} E^R [Y_T] = \sup_{\mathcal{M}} E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right]$, whence $A = \sup_{\mathcal{P}} E^R \left[Y_T - \int_0^T \lambda_t^* dS_t \right]$ and (2) via the previous lemma.

The last point to check is the admissibility of λ^*. By taking conditional expectations with respect to P in (3):

$$\int_0^t \lambda_u^* dS_u \geq E^P [Y_T \mid F_t] - A \geq -A \ \ a.s. \tag{4}$$

so that λ^* is admissible.

3 Existence of a minimizing multiplier

The crucial step is the existence of a minimizing multiplier. We shall make use of our Hilbert space context in order to give a proof of the following:

Theorem 5 *There is some λ^* in $\mathcal{H}$ satisfying (3).*

We shall need the two lemmatas:

Lemma 6 *Let $R \in \mathcal{P}$. Then $R \in \mathcal{M}$ if and only if $\lambda(R) = 0$ in $\mathcal{H}$.*

Indeed $R \in \mathcal{M}$ if and only if $SD(R)$ is a $P-$ martingale. Now

$$\begin{aligned} S_t D_t(R) &= S_t \left(1 + \int_0^t \lambda_u(R)\, dS_u + \theta(R)_t\right) \\ &= S_t + S_t \int_0^t \lambda_u(R)\, dS_u + S_t \theta(R)_t \end{aligned}$$

Since both $t \mapsto S_t$ and $t \mapsto S_t\theta(R)_t$ are $P-$martingales, this amounts to $t \mapsto S_t \int_0^t \lambda_u(R)\, dS_u$ being a martingale, or yet by the definition of the square bracket: $\left(\int_0^t \lambda_u(R)\, d[S]_u\right)_{0 \leq t \leq T} \equiv 0$. This is equivalent to $\lambda_u(R) = 0$ $d[S]_u\, dP\, a.s.$, or yet $\lambda(R) = 0$ in $\mathcal{H}$.

Lemma 7 *Let $R \in \mathcal{P}$, $\gamma \in \mathcal{H}$. Then if $< ., . >$ denotes the scalar product in $\mathcal{H}$:*

$$\begin{aligned} < \gamma, \lambda(R) > &= E^P\left[\int_0^T \gamma_t \lambda(R)_t\, d[S]_t\right] \\ &= E^R\left[\int_0^T \gamma_t dS_t\right] \end{aligned}$$

Indeed

$$\begin{aligned}
E^R\left[\int_0^T \gamma_t dS_t\right] &= E^P\left[\tfrac{dR}{dP}\int_0^T \gamma_t dS_t\right] \\
&= E^P\left[\left(1+\int_0^T \lambda\left(R\right)_t dS_t + \theta_T\right)\int_0^T \gamma_t dS_t\right] \\
&= E^P\left[\int_0^T \lambda\left(R\right)_t dS_t \int_0^T \gamma_t dS_t\right] \\
&= E^P\left[\int_0^T \lambda\left(R\right)_t \gamma_t d\left[S\right]_t\right]
\end{aligned}$$

Let us now turn to the proof of the theorem.

3.1 Temptative proof

We follow very closely Föllmer and Kabanov's line of reasoning. Consider the following set of $\mathbb{R}\times\mathcal{H}$:

$$C = \left\{(x,\beta)\ /\ \exists R\in\mathcal{P},\, x < E^R\left[Y_T\right], \beta = \lambda\left(R\right)\ \right\}$$

Then the set C is easily seen to be a non-empty convex set. Moreover $(A,0)$ does not belong to C because of lemma 6. Assume now that there is a non-zero *continuous* linear form which separates C and $(A,0)$. Then since $\mathbb{R}\times\mathcal{H}$ is an Hibert space, for some (a,α) in $\mathbb{R}\times\mathcal{H}$ and any (x,β) in C :

$$ax + <\alpha,\beta> \leq aA$$

Now if (x,β) belongs to C, so does (y,β) for $y<x$, which entails $a\geq 0$. If $a>0$, then

$$x + <\frac{\alpha}{a},\beta> \leq A$$

which gives by continuity as $x\uparrow E^R\left[Y_T\right]$ for $R\in\mathcal{P}$:

$$E^R\left[Y_T\right] + <\frac{\alpha}{a},\beta> \leq A$$

or yet by setting $\lambda^* = -\frac{\alpha}{a}$:

$$E^R[Y_T] - < \lambda^*, \beta > \leq A$$

or yet yet by lemma 7

$$E^R[Y_T] - E^R\left[\int_0^T \lambda_t^* dS_t\right] \leq A$$

for any R in $\mathcal{P}$, whence the result by the end of the proof of lemma 4.

What happens in case $a = 0$? Then since (a, α) is not zero, $< \alpha, \lambda(R) > \leq 0$ for every R in $\mathcal{P}$ and some $\alpha \neq 0$.

This means for every R in $\mathcal{P}$

$$E^R\left[\int_0^T \alpha_t dS_t\right] = 0$$

or yet by the same reasoning as in the proof of lemma 4, $\int_0^T \alpha_t dS_t = 0$ almost surely, which gives $\left(\int_0^T \alpha_t dS_t\right)^2 = 0$ almost surely and since α is in $\mathcal{H}$, $\alpha = 0$, whence a contradiction.

3.2 Proof

In order to get a *continuous* separating linear form, we need the property that C contains at least an internal point with respect to the ambient vector space (i.e. some point a such that for any vector b of the ambient vector space, $a+\varepsilon b$ belongs to C for ε small enough-which may well depend on b).

A way to get this property is to replace C by the following set: let

$$C_1 = \left\{ \begin{array}{c} (x, \beta) \in \mathbb{R}\times\mathcal{H} \,/\, \exists\theta \in \Theta,\, \theta_T > -1\, a.s.,\, E^P[\theta_T] = 0, \\ x < E^P\left[Y_T\left(1 + \int_0^T \beta_t dS_t + \theta_T\right)\right] \end{array} \right\}$$

Then C_1 is a non-empty (in fact it contains C) convex set.

Moreover, R belongs to $\mathcal{M}$ if and only if $E^R[Y_T] = E^P[Y_T(1+\theta_T)]$ for some θ in Θ satisfying $\theta_T > -1\, a.s.$, $E^P[\theta_T] = 0$, so that $(A, 0)$ does not belong to C_1.

This time it is obvious that every point of C_1 belongs to its interior for the strong topology, therefore is an internal point, so that there is a continuous linear form separating $(A, 0)$ and C_1.

Now observe that we can conclude as above, since our new convex set contains the previous one.

4 Conclusion: relation to the optional decomposition

As a conclusion, let us stress that our approach yields a weaker result than the optional decomposition theorem: if we consider the process

$$Z_t = ess \sup_{M_e(Q)} E\left[Y_T \mid F_t\right]$$

we can assert for λ^* a minimizing multiplier that

$$Z_0 + \int_0^T \lambda_t^* dS_t - Z_T \geq 0 \, a.s.$$

By conditioning it is straightforward to get

$$Z_0 + \int_0^t \lambda_u^* dS_u - Z_t \geq 0 \ a.s.$$

but we do *not* know the sign of

$$Z_t + \int_t^T \lambda_u^* dS_u - Z_T$$

for other values of t between 0 and T, whereas the optional decomposition of Z (theorem 2) satisfies

$$Z_t + \int_t^T \Delta_u dS_u - Z_T = A_T - A_t \geq 0$$

In other words, a strategy associated to an optional decomposition has the nice following feature: if you sell the option at time $t > 0$ at price Z_t, then you can follow the same (super)-hedging policy as the one you would follow at that time if you had sold the the option at time 0 at price Z_0. Another way to say the same thing is the following: if you reevaluate your strategy at time t with an option price given by Z_t, then your current Profit&Loss will be non-negative. Now in case of an "optional" strategy, if you look at the component of your Profit&Loss between time t and maturity, it will also be non-negative, whereas it might well fell below zero for an arbitrary minimizing λ^*-even if the final Profit&Loss at maturity will be non-negative in both cases.

This property does not hold in general for an arbitrary choice of a minimizing multiplier λ^*. In fact, should the optional decomposition enjoy a uniqueness property, then this property is of course a characterizing one. We shall now design an example where there is a unique optional decomposition whereas there are plenty of minimizing multipliers.

Notice that in a discrete-time context it is easy to design situations like the following one in a two-period trinomial model, for instance.

The uniqueness of the optional decomposition holds if the increasing process A_t happens to be predictable, in which case it amounts to that of the Doob's decomposition. This is the case if the filtration F is that of some finite-dimensional Brownian motion (cf [ElKQ], and also [K] lemma 2.1 for a generalization).

So let us design an example where there are many processes which satisfy (3):

Let $(B(1), B(2))$ a planar Brownian motion under Q. Let σ a bounded continuous injective function with values in $[1,2]$ and assume that S is given by:

$$dS_t = \sigma\left(B(2)_t\right) S_t dB(1)_t\,,\ S_0 > 0$$

Then it is clear that the filtration generated by S is that of $(B(1), B(2))$ and also thanks to the Girsanov theorem that the equivalent martingale measures are given by those which takes $B(2)$ to a Brownian motion with a drift $t \mapsto \int_0^t \lambda_u du$ where λ is predictable with (at least) $\int_0^t \lambda_u^2 du$ finite almost surely.

Consider now the quantity

$$X_T = \int_0^{T \wedge \tau_1} e^{-\left|B(2)_t\right|} dB(1)_t \tag{5}$$

where τ_1 is the first time where $\left|\int_0^t e^{-|B(2)_u|} dB(1)_u\right|$ is greater than the level 1. Then for any R in $M_e(Q)$, $E^R[X_T] = 0$ or yet $E^R\left[X_T^+\right] = E^R\left[X_T^-\right]$.

It is also easy to see that $\inf_{R \in M_e(Q)} E^R\left[X_T^{\pm}\right] = 0$.

Take then (for instance) any stopping time τ less than $T \wedge \tau_1$. Set

$$\mu_t = e^{-|B(2)_t|} 1(t < \tau \wedge T \wedge \tau_1),\ \nu_t = e^{-|B(2)_t|} 1(\tau < t < T \wedge \tau_1)$$

Then:

$$\int_0^T \mu_t dB(1)_t - X_T^+ = \int_0^T (-\nu)_t \, dB(1)_t - X_T^-$$

Set now for any real number a greater than 2

$$\begin{aligned} Y_T^a &= a + \int_0^T \mu_t dB(1)_t - X_T^+ \\ &= a + \int_0^T (-\nu)_t \, dB(1)_t - X_T^- \end{aligned}$$

Then Y_T^a is positive and bounded, (H) and $(H1)$ are in force. Moreover for any R in $M_e(Q)$, $E^R[Y_T^a] = a - E^R\left[X_T^+\right]$ so that

$$\sup_{M_e(Q)} E^R[Y_T^a] = a - \inf_{R \in M_e(Q)} E^R\left[X_T^+\right] = a$$

Now both processes $t \mapsto \frac{\mu_t}{\sigma\left(B(2)_t\right) S_t}$ and $t \mapsto \frac{\nu_t}{\sigma\left(B(2)_t\right) S_t}$ yield superstrategies.

Of course, the choice of (5) is a bit artificial, but by replacing $|B(2)_t|$ by (for instance), $\int_0^t |B(2)_u| \, du$ and integrating by parts one gets a pathwise defined quantity-this way one can design more realistic payoff functions Y_T^a.

Let us note also that the optional decomposition theorem allows to deal with American contingent claims by looking at the adequate $R-$supermartingale for any R in $M_e(Q)$, whereas the way to adapt our proof to the American case is not clear.

Finally, let us turn to an application to the characterization of attainable claims (cf [AS], [ElKQ]):

4.1 Characterization of attainable claims

Introduce now the program which corresponds to the buyer of the option point of view:

$$c^* = \sup \left\{ c \in \mathbb{R} \,/\, \exists\, \Delta \text{ admissible, } c - \int_0^T \Delta_t dS_t \leq Y_T \ a.s. \right\} \tag{6}$$

Buy following exactly the same line of reasoning we get the existence of a λ^* in $\mathcal{H}$ such that with $B = \inf_{R \in M_e(Q)} E^R [Y_T]$:

$$B - \int_0^T \lambda_t^* dS_t \leq Y_T \ a.s.$$

By taking conditional expectations with respect to P :

$$B - \int_0^t \lambda_u^* dS_y \leq E^P [Y_T \mid F_t] \ a.s.$$

whence $\int_0^t \lambda_u^* dS_y \geq B - \|Y_T\|_\infty$ a.s. where $\|Y_T\|_\infty$ is the $L^\infty (P)$ norm of Y_T (note that in the same way the boundedness of Y_T may be used in place of the assumption $Y_T \geq 0$ a.s. in the derivation of (4), so that the assumption $Y_T \geq 0$ a.s. may be dropped). Therefore λ^* is admissible, whence:

Theorem 8 *In (6) the supremum is attained and*

$$c^* = \inf_{R \in M_e(Q)} E^R [Y_T]$$

Let us now set the following:

Definition 9 *A bounded F_T-measurable Y_T is said to be attainable if there exists a real number c and a process Δ with both Δ and $-\Delta$ are admissible such that*

$$c + \int_0^T \Delta_t dS_t = Y_T \ a.s. \tag{7}$$

Then we get the characterization of attainable random variables:

Proposition 10 *Y_T is attainable if and only if the application $R \mapsto E^R[Y_T]$ is constant over $M_e(Q)$.*

Indeed, if Y_T is attainable, then with c in (7) $c = E^R[Y_T]$ for any R in $M_e(Q)$ since the process $t \mapsto \int_0^t \Delta_u dS_u$, which is bounded due to the admissibility assumptions, is a true $R-$martingale. For the converse way, let $c = E^P[Y_T]$. Then by theorems 1 and 8 there are some $\lambda^*(+)$, $\lambda^*(-)$ in $\mathcal{H}$ such that

$$c + \int_0^T \lambda^*(+)_t \, dS_t \geq Y_T \geq c + \int_0^T \lambda^*(-)_t \, dS_t \ a.s.$$

In particular $\int_0^T \lambda^*(+)_t \, dS_t \geq \int_0^T \lambda^*(-)_t \, dS_t$ a.s.. But since $\lambda^*(+)$ and $\lambda^*(-)$ are in $\mathcal{H}$ this entails $\lambda^*(+) = \lambda^*(-)$ in $\mathcal{H}$ whence $\int_0^T \lambda^*(+)_t \, dS_t = \int_0^T \lambda^*(-)_t \, dS_t$ a.s. and finally

$$c + \int_0^T \lambda^*(+)_t \, dS_t = Y_T = c + \int_0^T \lambda^*(-)_t \, dS_t \ a.s.$$

whence the result.

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